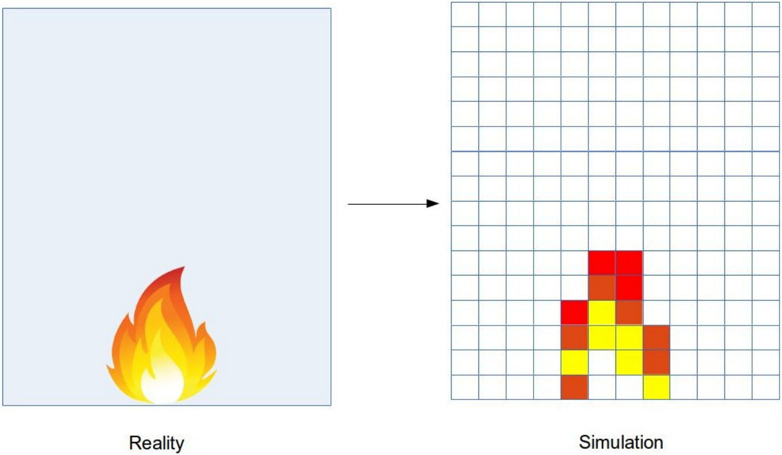
dgfire

Smoke and fire simulations with *hp*-adaptive Discontinuous Galerkin Methods

April 8, 2014 | Kevin Drzycimski | JSC/Research Centre Jülich

Smoke and fire simulation



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Part I: What's wrong with FDS?

April 8, 2014 | Kevin Drzycimski

FDS

State of the art

The Fire Dynamics Simulator (FDS) from the National Institute of Standards and Technology (NIST) is todays most common fire simulation software.

Pros

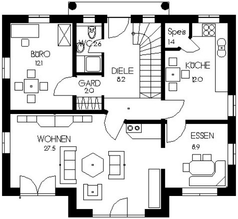
- free, open-source
- well established
- large feature set

FDS

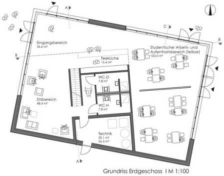
Cons

Geometries

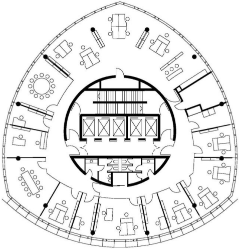
- Finite Difference Method, simple implementation
- only rectangular geometries are represented exact



OK



awkward



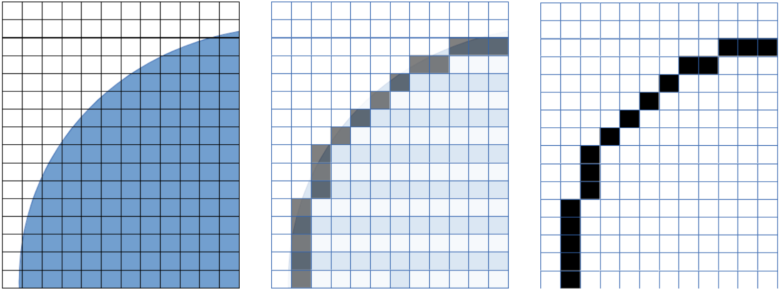
impossible

FDS

Cons

Staircase

- Rasterization of non-rectangular geometries
- leads to different results



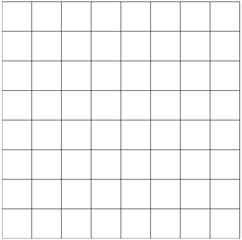
round shape rasterization staircase-effect

FDS

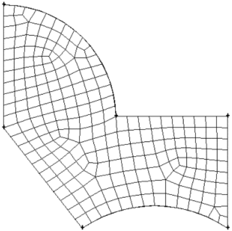
Cons

Triangulations

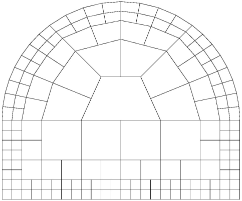
- equal-sized grid cells
- problems with multiple grids



equidistant



irregular

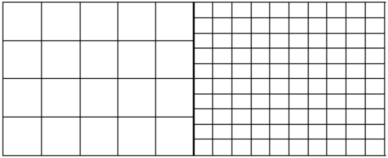


hanging nodes

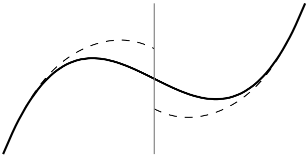
FDS
Cons

Distributed Computing

- Pressure is solved only in local compartments
- serial and parallel computations differ



different meshes



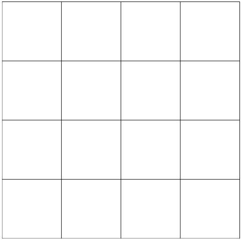
discontinuous pressure

FDS

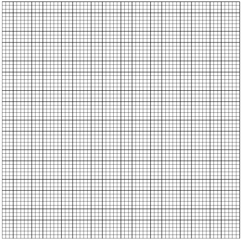
Cons

Adaptive Mesh Refinement

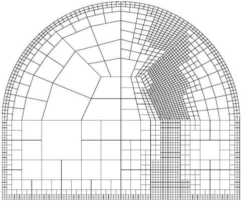
- Grids are static
- Need to chose optimal grid size



too big



too small



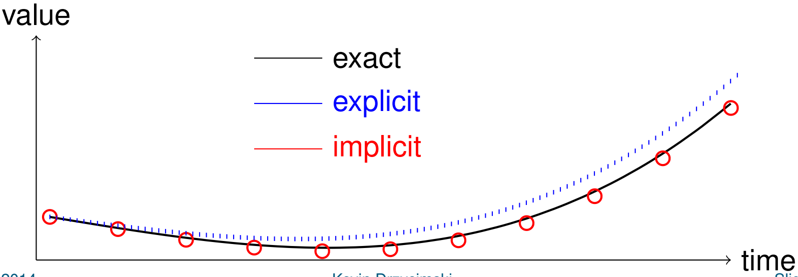
optimal?

FDS

Cons

Explicit time stepping

- Heavy restrictions on stepsize
- Stability?



FDS

Summary

Pros

- free, open-source
- well established
- large feature set

Cons

- finite difference scheme
- explicit time stepping
- no spatial adaptivity
- not suitable for parallelization

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Part II: Discontinuous Galerkin

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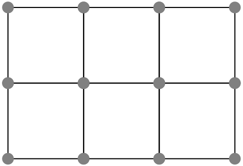
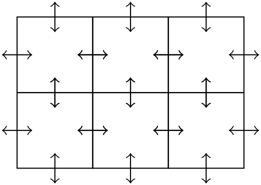
Discontinuous Galerkin

Introduction



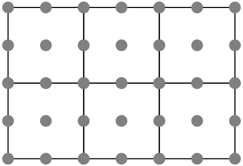
Finite Differences

- discretize on mesh vertices
- direct approximation of operators



Finite Volumes

- cell-centered, discontinuous values
- flux integrals on faces

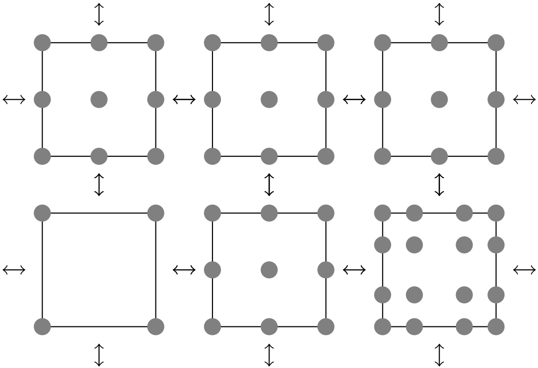


Finite Elements

- multiple values per cell
- high order polynomials
- continuous on faces

Discontinuous Galerkin

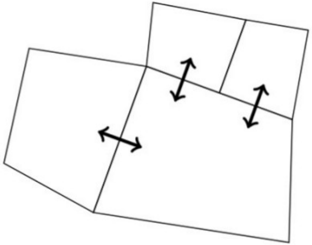
FEM + FVM

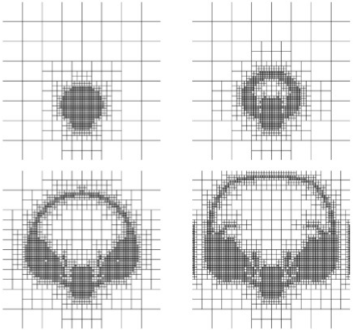


- FEM inside the cells: choose suitable order
- FVM between the cells: flux conservation

Discontinuous Galerkin

h-adaptive

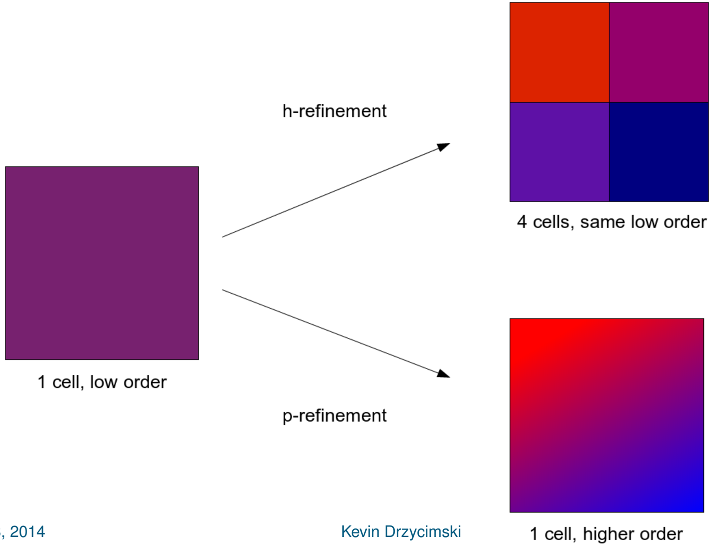




- Allow different refinement levels
- Change resolution according to solution

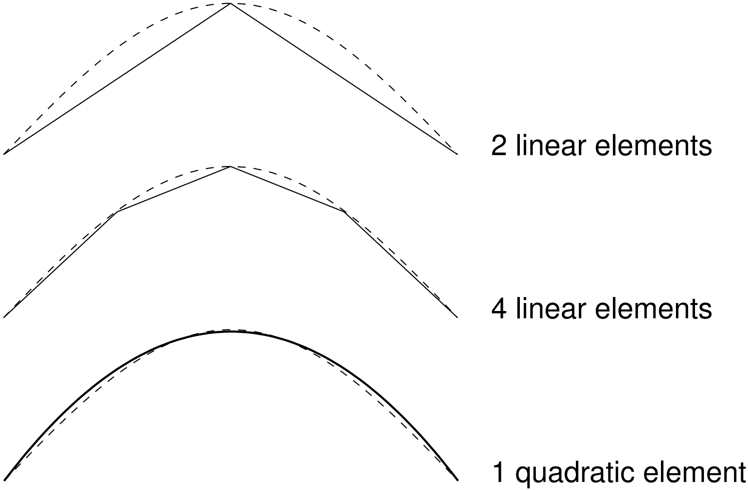
Discontinuous Galerkin

hp-adaptive



Discontinuous Galerkin

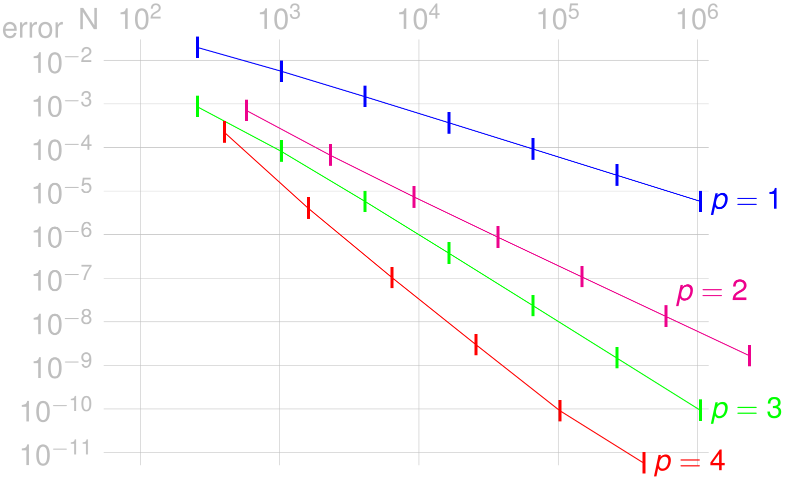
High Order



Discontinuous Galerkin

High Order

$-\Delta u = f, \quad u_{exact} = e^{-||x||^2}$, global refinement



Discontinuous Galerkin

Summary

Pros

- discrete preservation of conservation variables
- order of error reduction may differ locally
- unstructured grids (with hanging nodes)
- straightforward parallelization

Cons

- complex theory
- expensive implementation

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Part III: Computational Fluid Dynamics

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Fluid dynamics
Equations

- Incompressible Navier-Stokes Equations

$$\frac{\partial u}{\partial t} + u \cdot \nabla u - \mu \Delta u + \nabla p = f(T)$$
$$\nabla \cdot u = 0$$

- Convection-Diffusion for Temperature

$$\frac{\partial T}{\partial t} + \underbrace{u \cdot \nabla T}_{\textit{convective}} - \underbrace{\kappa \Delta T}_{\textit{diffusive}} = \gamma$$

Fluid dynamics
Algorithm

Chorin-Projection:

$$(u^n, T^n) \xrightarrow{\mathrm{I}} u^* \xrightarrow{\mathrm{II}} (u^{n+1}, p^{n+1}) \xrightarrow{\mathrm{III}} T^{n+1}$$

$$\frac{1}{\delta t} u^* + u^* \cdot \nabla u^* - \mu \Delta u^* = \frac{1}{\delta t} u^n + f(T^n) \qquad \mathrm{I}$$

$$-\Delta p^{n+1} = -\frac{1}{\delta t} \nabla \cdot u^* \qquad \mathrm{II}^{\mathrm{a}}$$

$$u^{n+1} = u^* - \delta t \nabla p^{n+1} \qquad \mathrm{II}^{\mathrm{b}}$$

$$\frac{1}{\delta t} T^{n+1} + u^{n+1} \cdot \nabla T^{n+1} - \kappa \Delta T^{n+1} = \frac{1}{\delta t} T^n + \gamma \qquad \mathrm{III}$$

Fluid dynamics

$$-\Delta p = f$$

- Finite Differences:

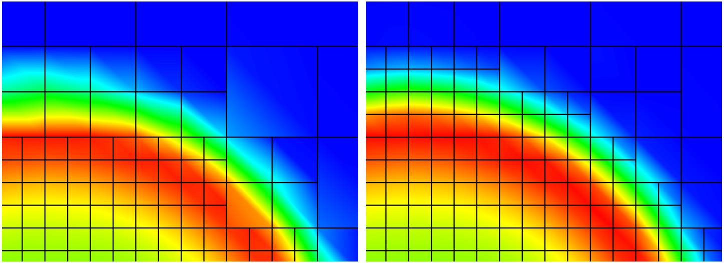
$$6p_{i,j,k} - p_{i\pm1,j\pm1,k\pm1} = h^2 f_{i,j,k}$$

- Discontinuous Galerkin:

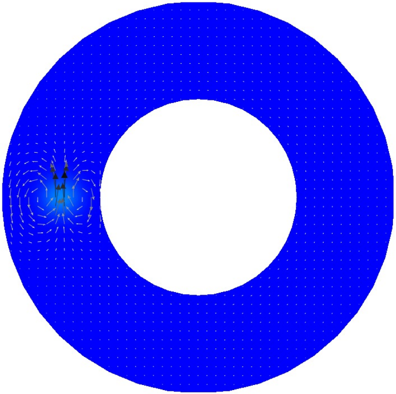
$$\int_\Omega \nabla p \nabla \varphi + \int_\Gamma -\{\nabla p\}[\![\varphi]\!] - [\![p]\!]\{\nabla \varphi\} + \eta [\![p]\!][\![\varphi]\!] = \int_\Omega f\varphi$$

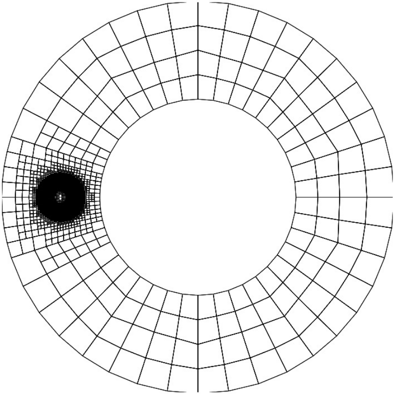
Fluid dynamics
Adaptive Refinement/Coarsening

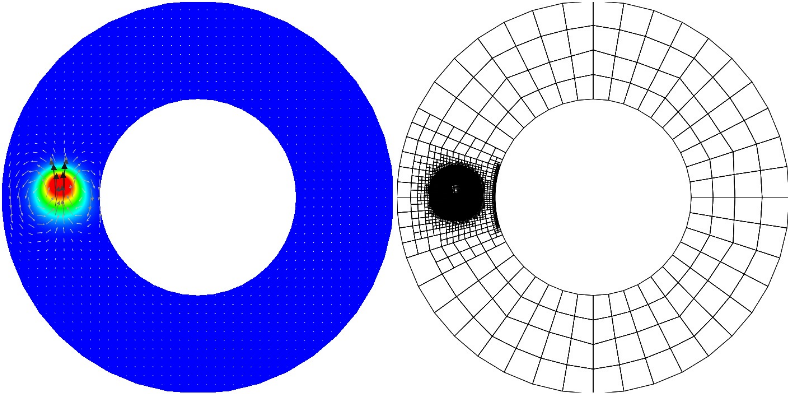
- $\nabla u, \nabla T$ for each cell $\rightarrow e_u, e_T$
- refine if $e_u > t_{refine}$ or $e_T > t_{refine}$
- coarse if $e_u < t_{coarse}$ and $e_T < t_{coarse}$

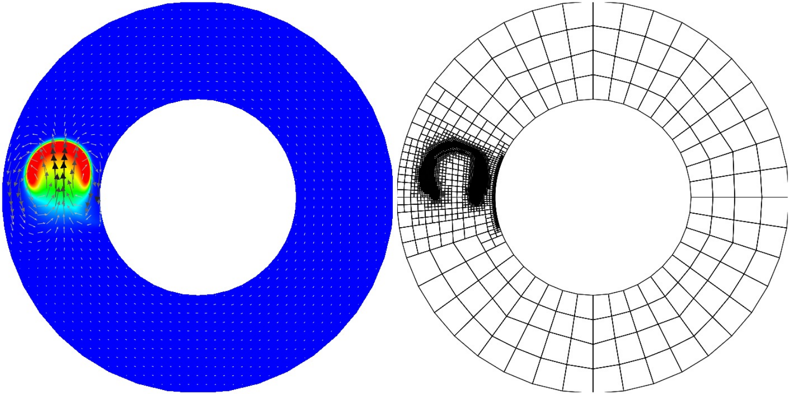


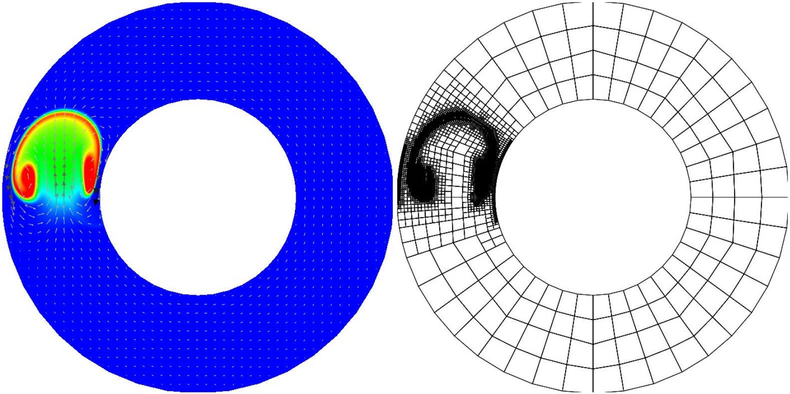
Member of the Helmholtz-Association

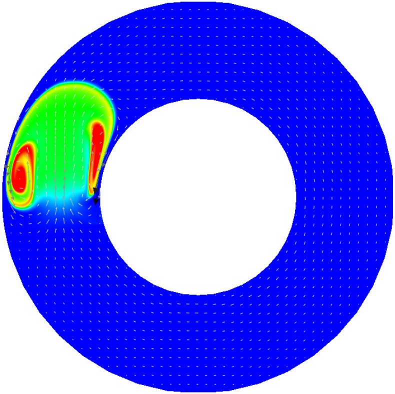


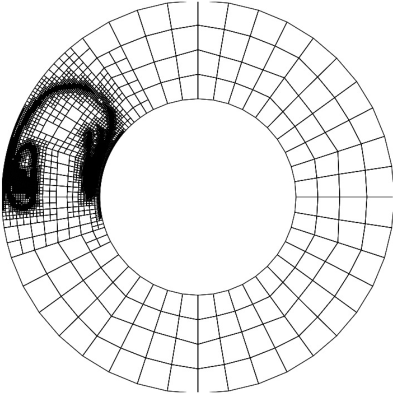


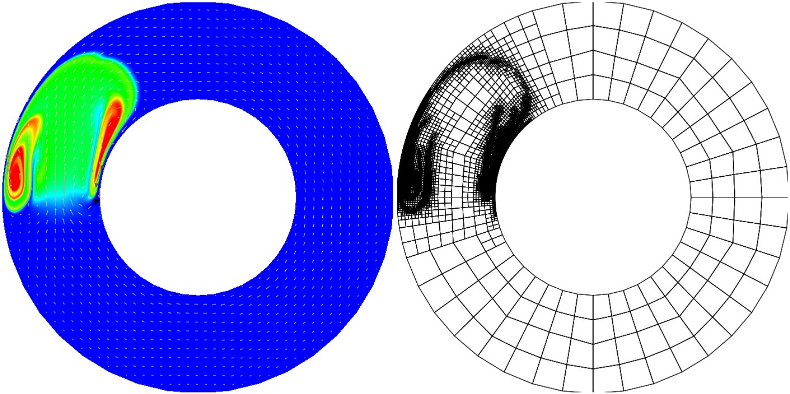


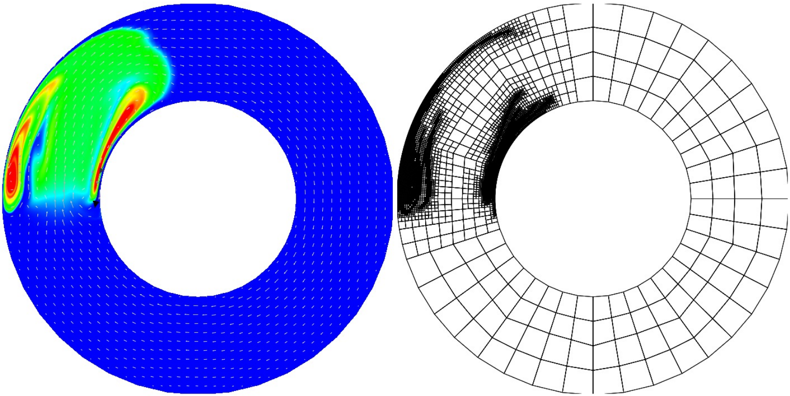


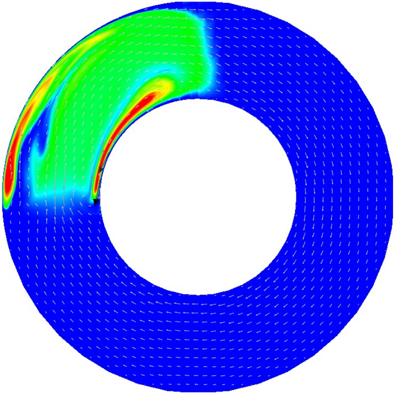


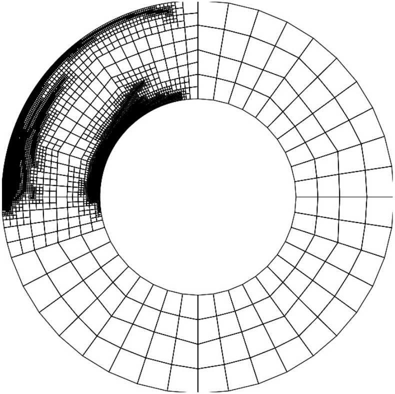


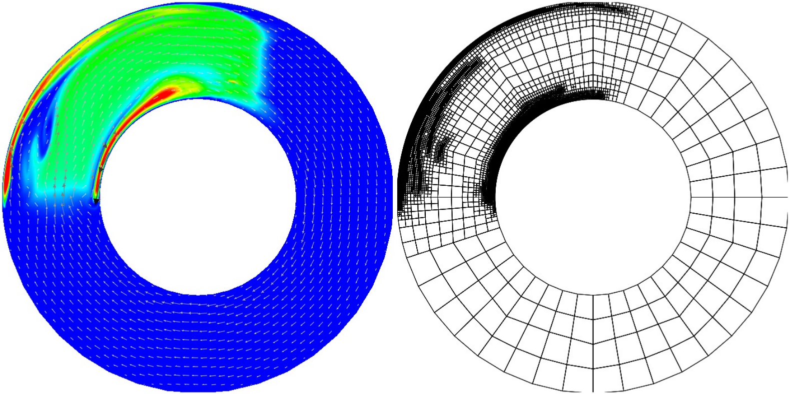


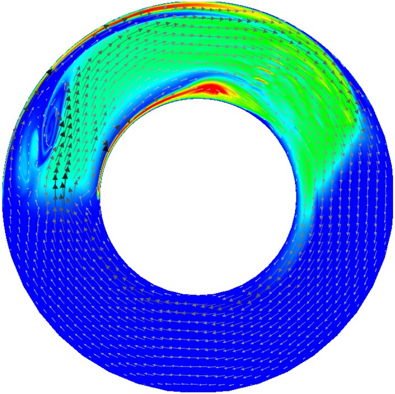


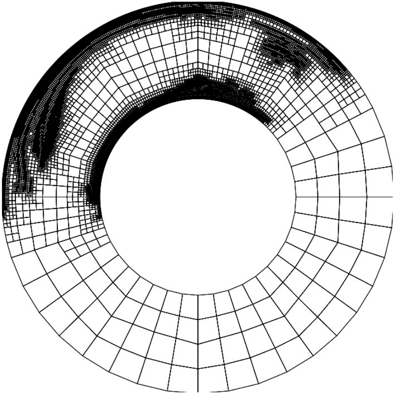


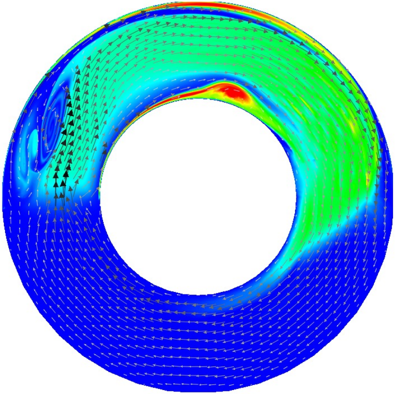


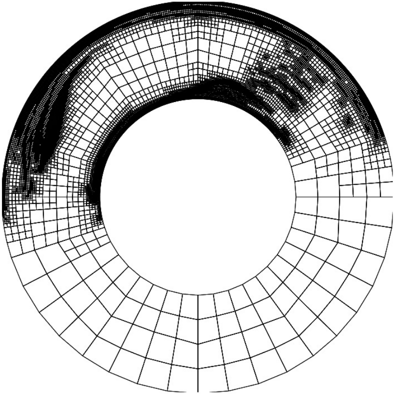


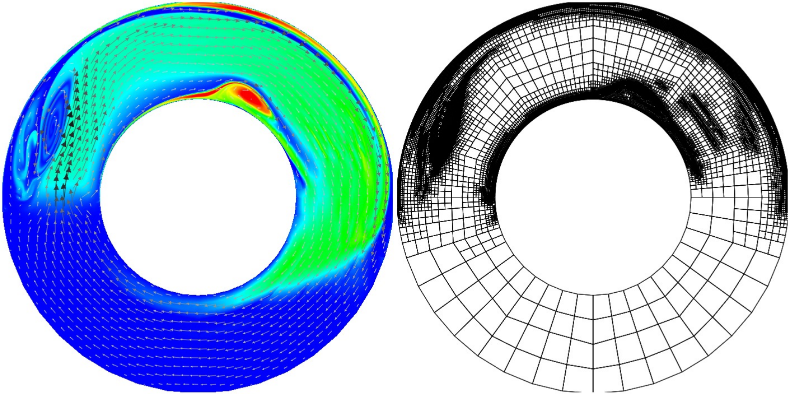


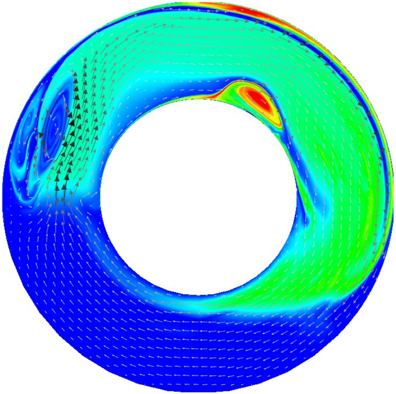


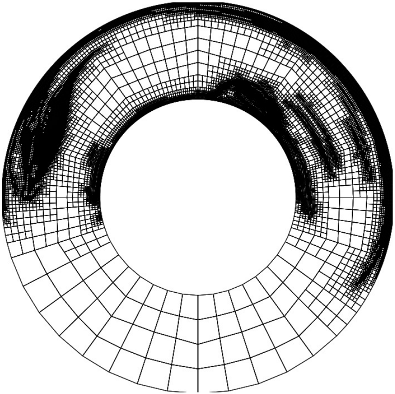


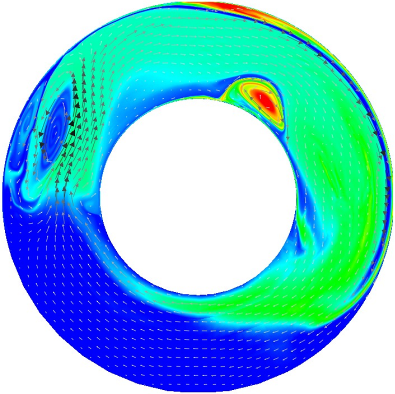


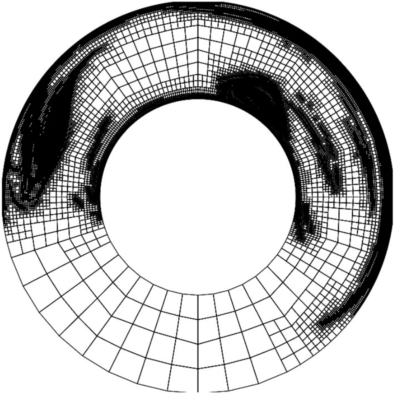


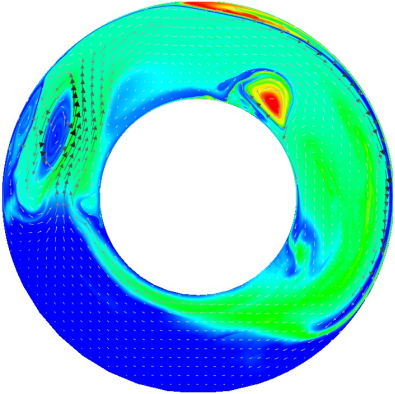


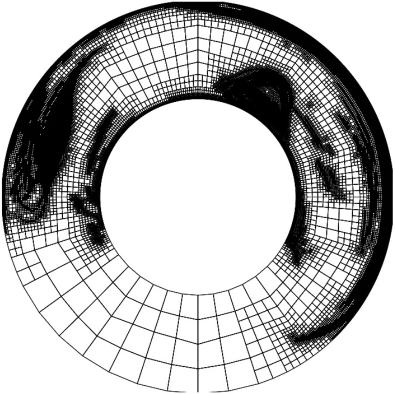


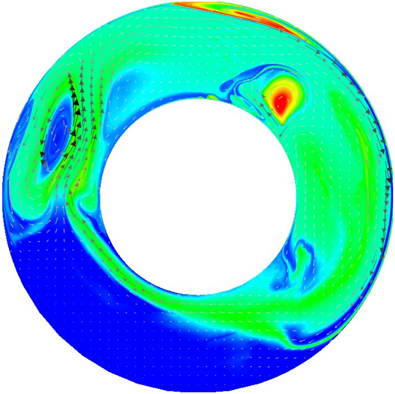


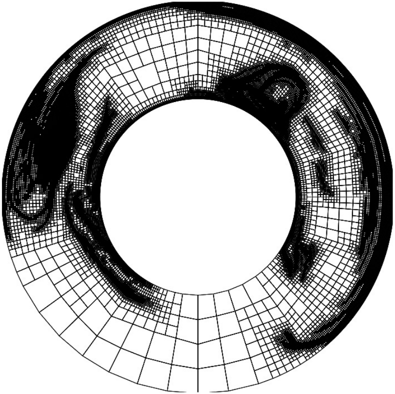


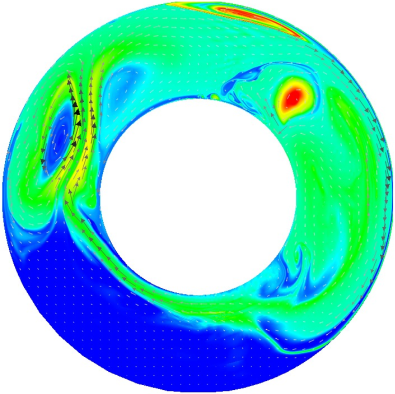


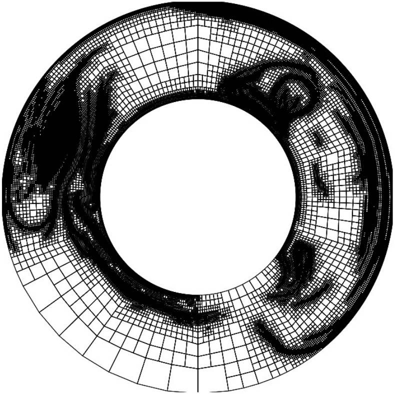


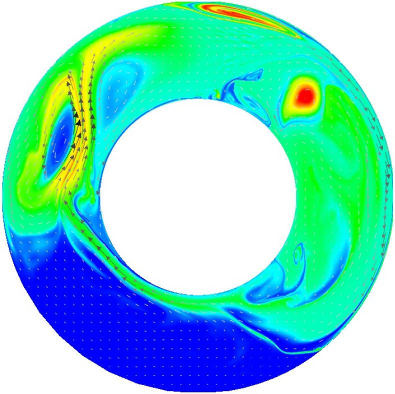


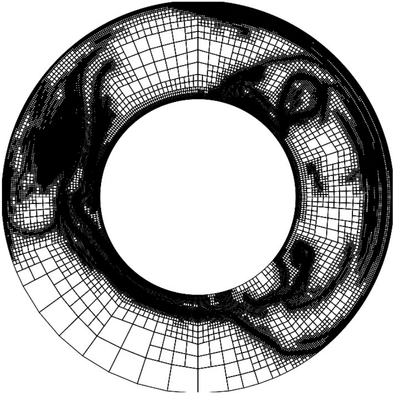


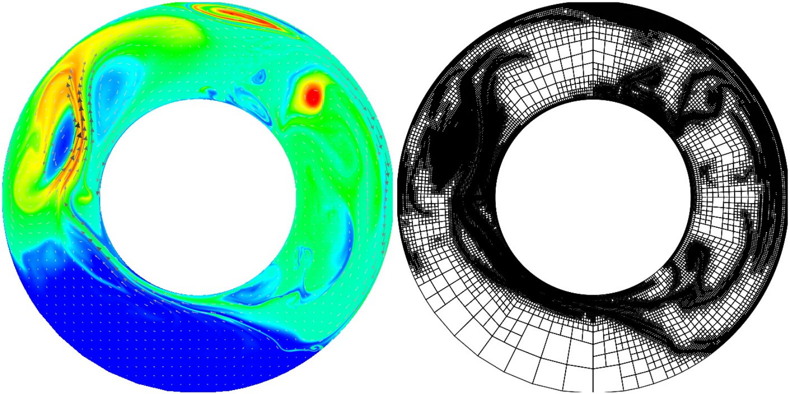


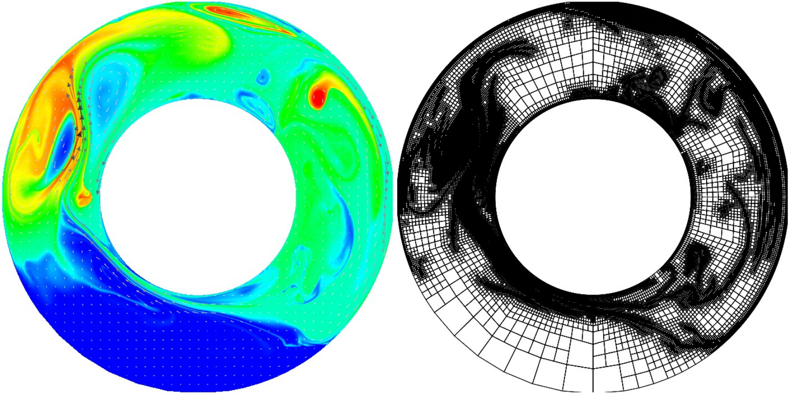


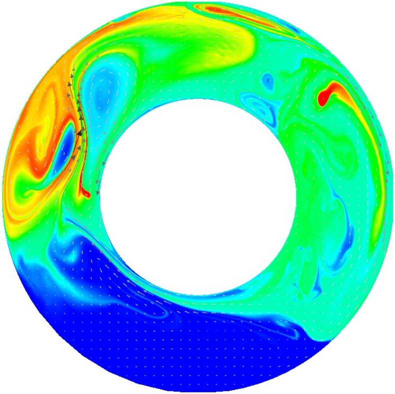


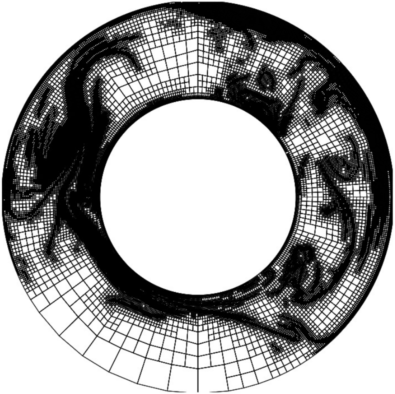


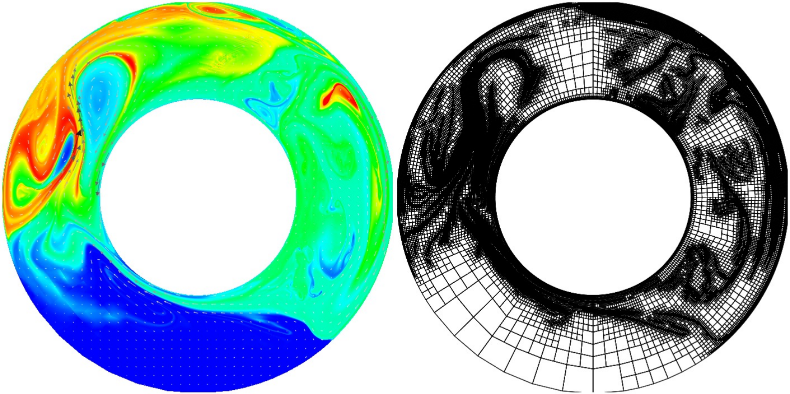


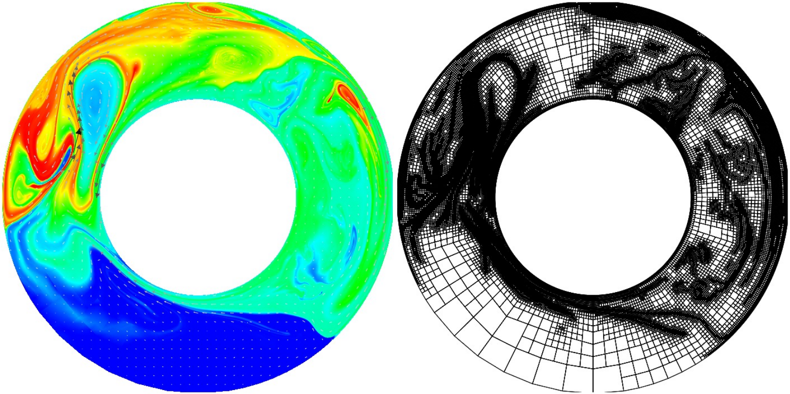


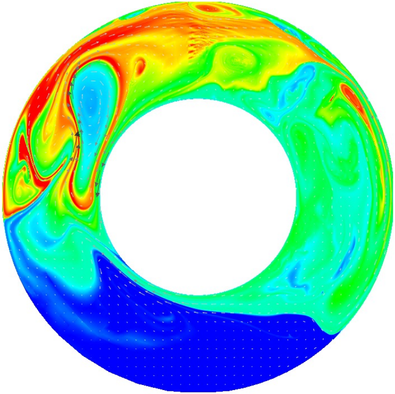


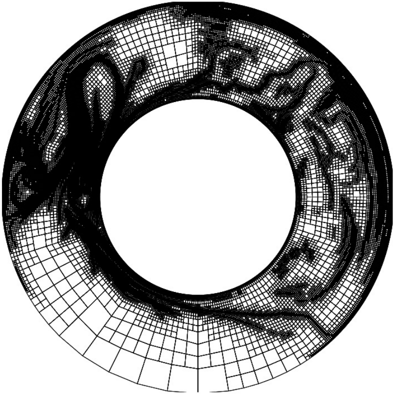


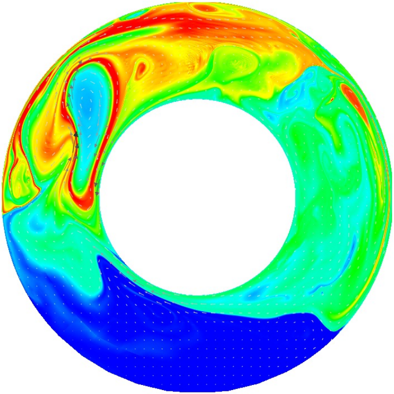


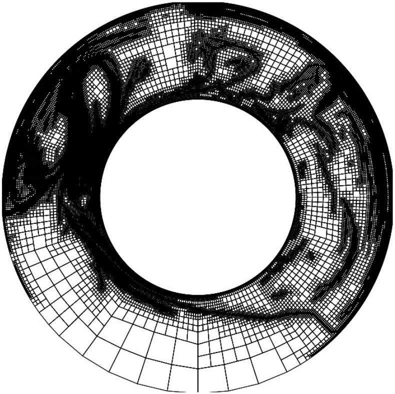


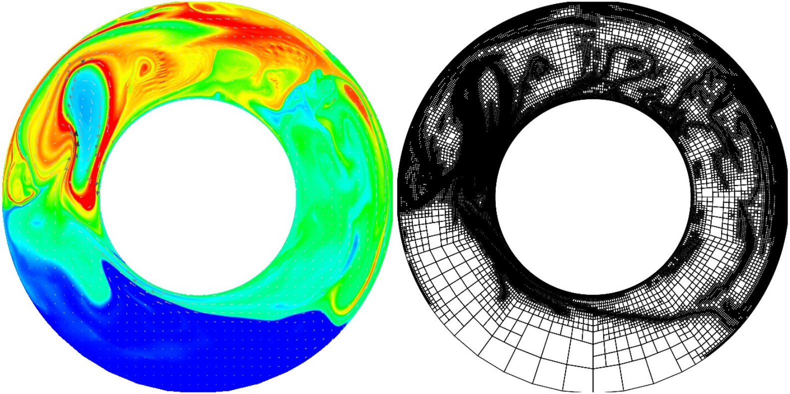


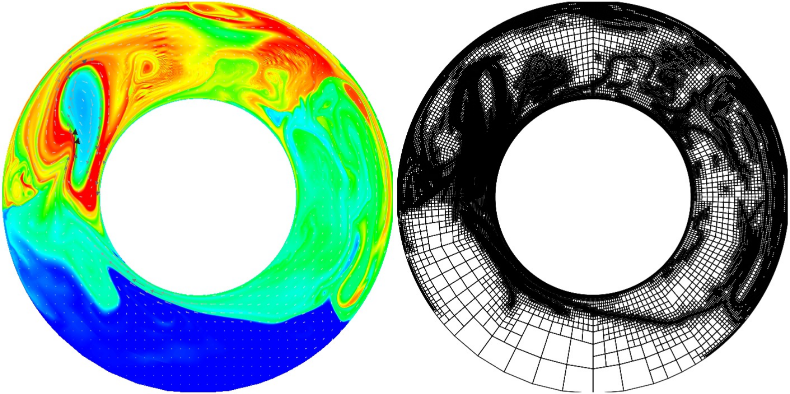


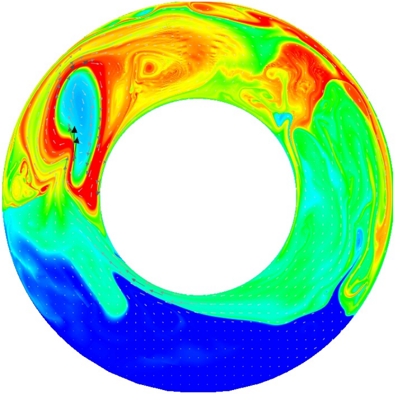


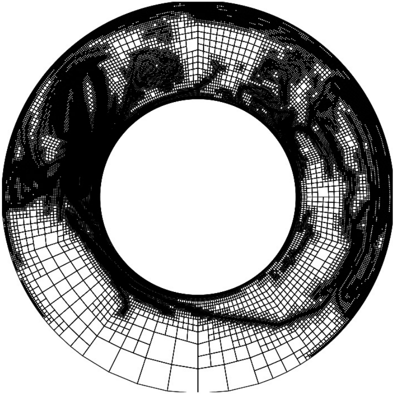


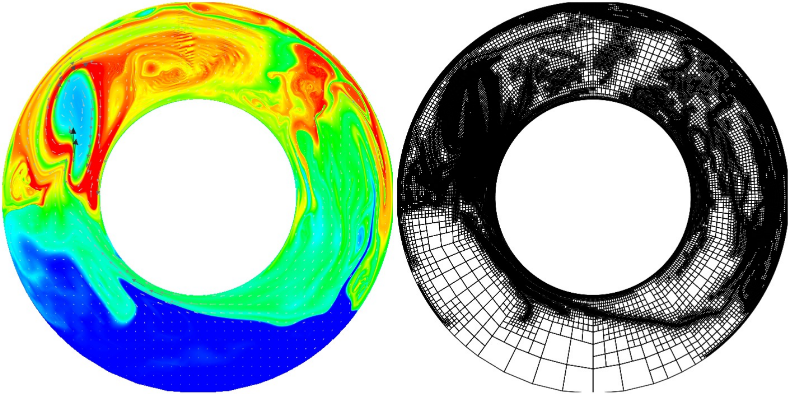


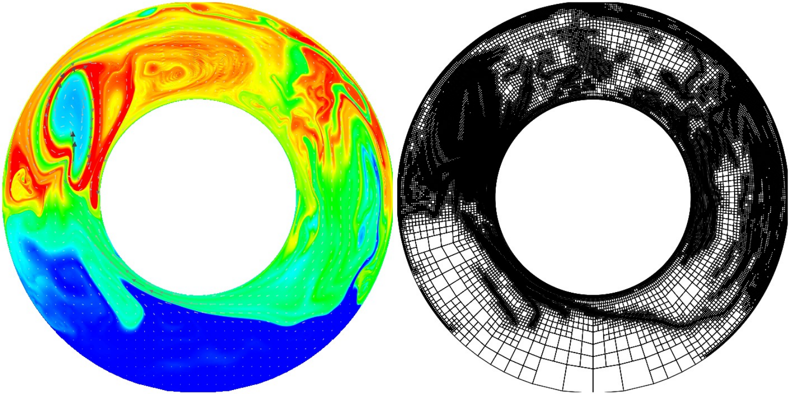


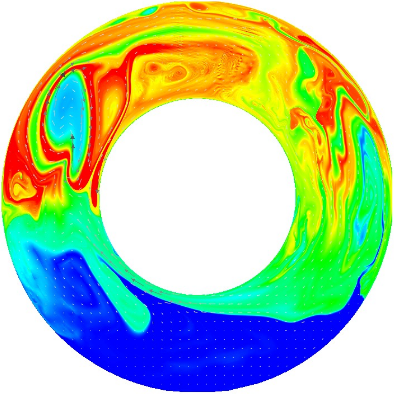


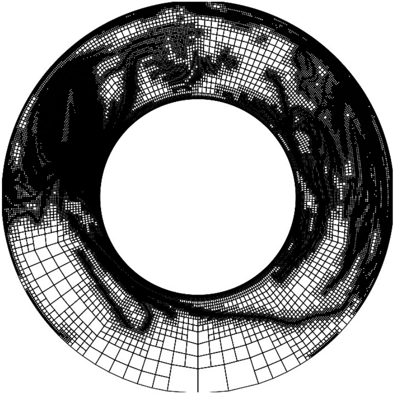


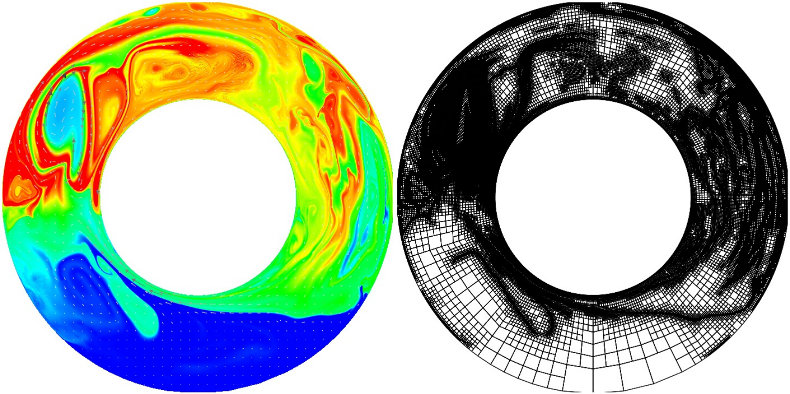


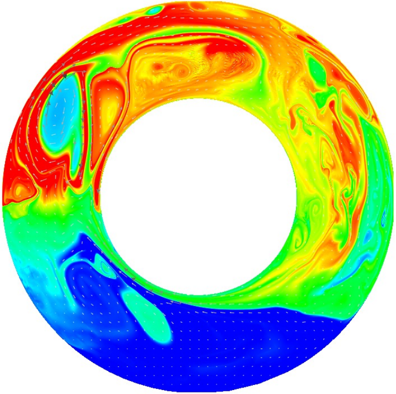


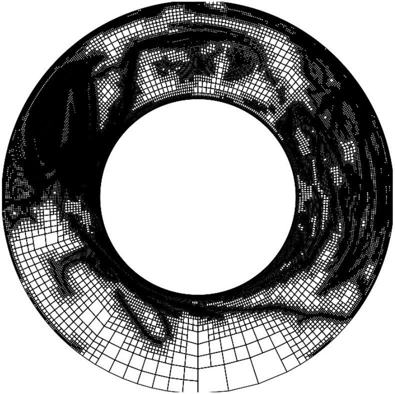












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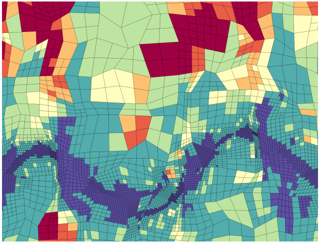
Part IV: Support Libraries

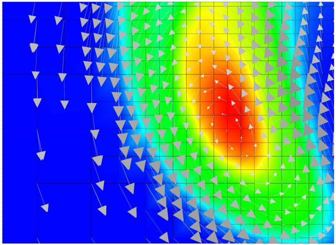
April 8, 2014 | Kevin Drzycimski

Support Libraries
Motivation

Needs

- Handling of adaptivelly refined grids
- Evaluation of integrals like $\int_K \tilde{u} \cdot \nabla v u \, dx$
- Parallelization, load balancing
- Solving ill-conditioned linear systems



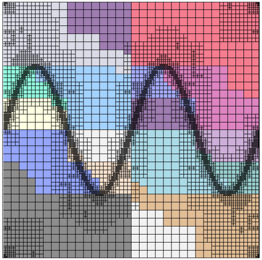


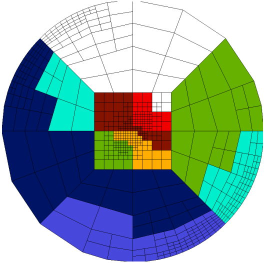
deal.II

Differential Equations Analysis Library

- extensive tutorial and documentation
- unified interface for 1D, 2D and 3D
- many types of Elements
- scales up to 16.000 processors
- `www.dealii.org`

p4est



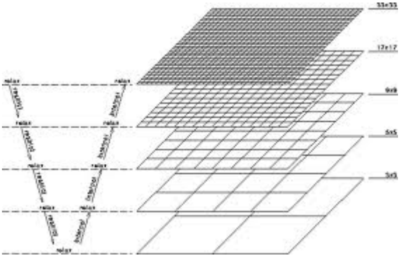


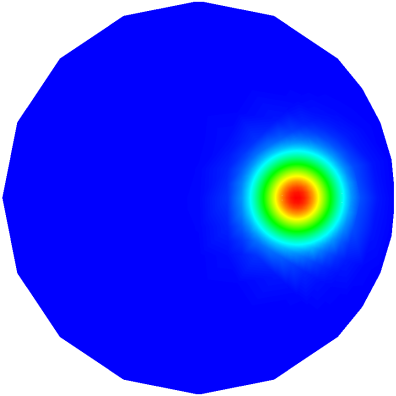
Parallel AMR on Forests of Octrees

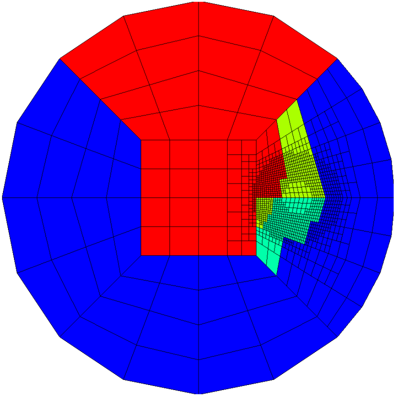
- Quadtrees in 2D, Octrees in 3D
- scales up to 100.000 processors
- `www.p4est.org`

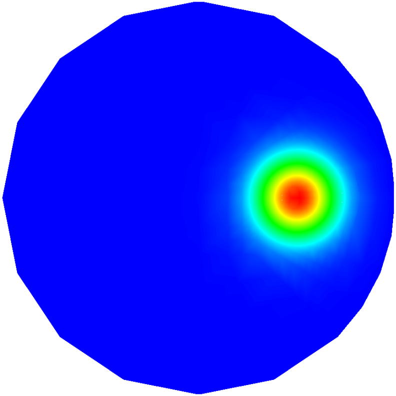
Trilinos

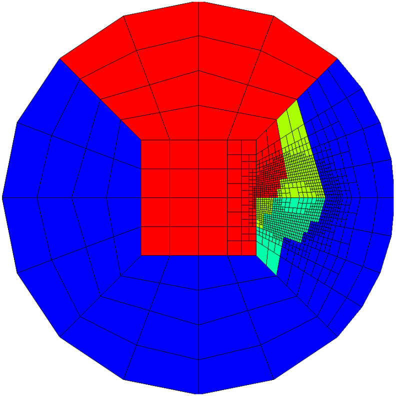
- Collection of scientific libraries
- using Linear Algebra Packages
- Multigrid Solver ML
- `www.trilinos.org`

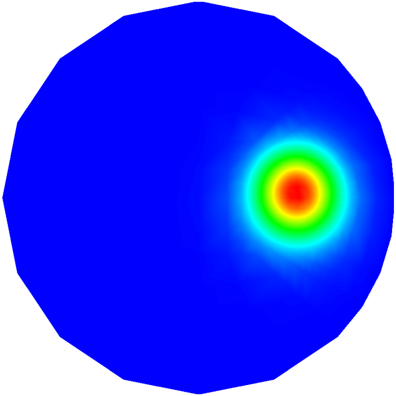


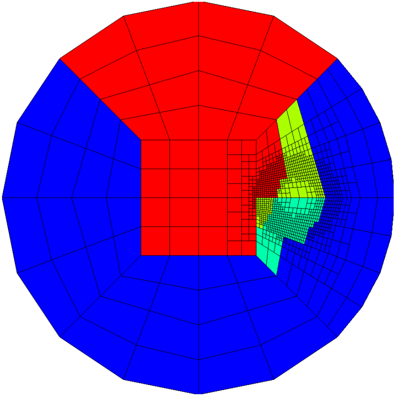


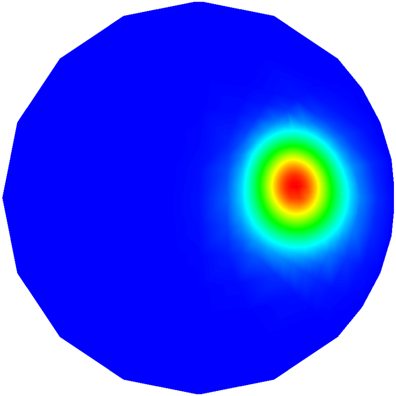


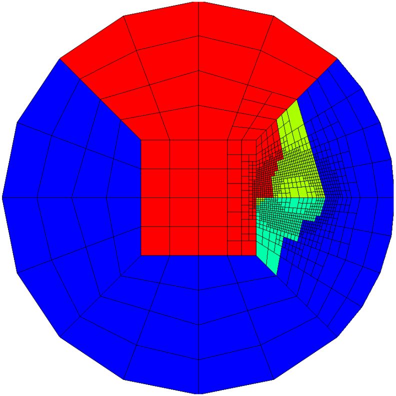


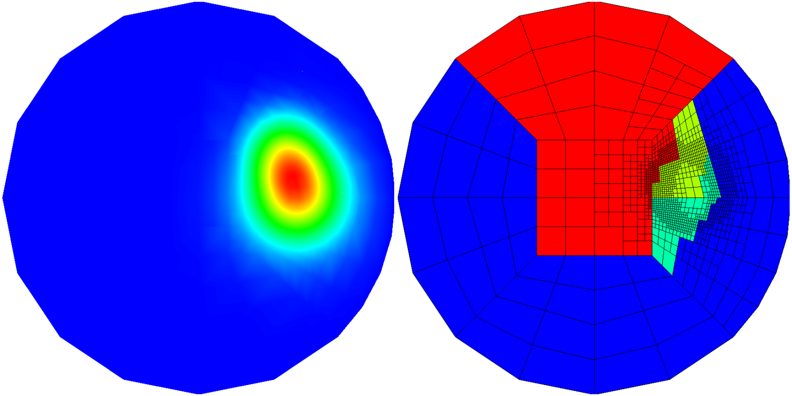


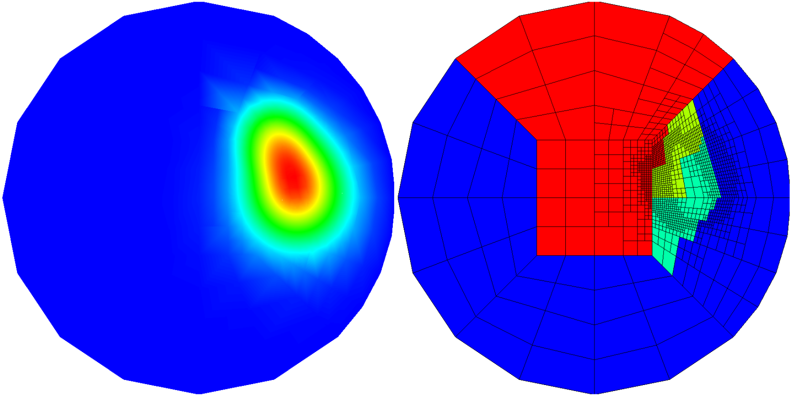


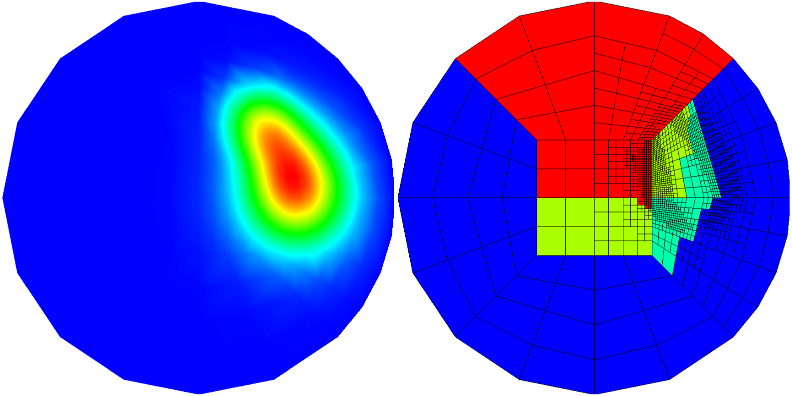


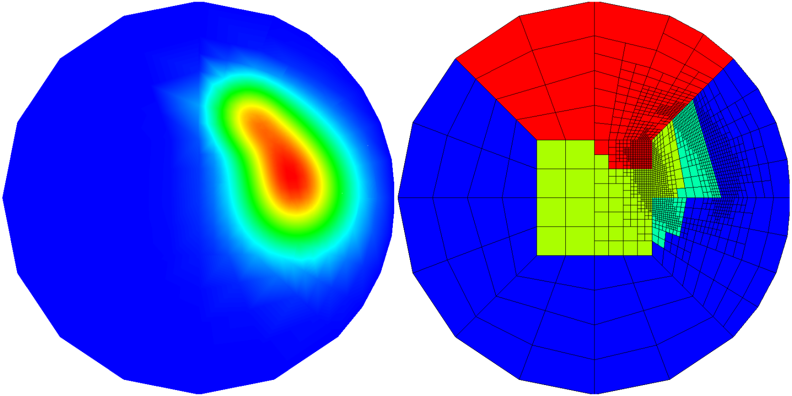


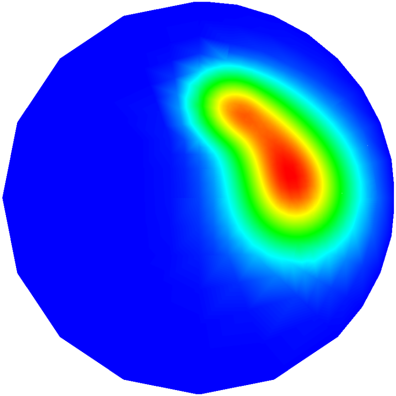


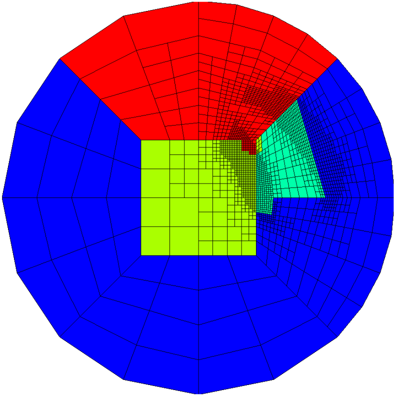


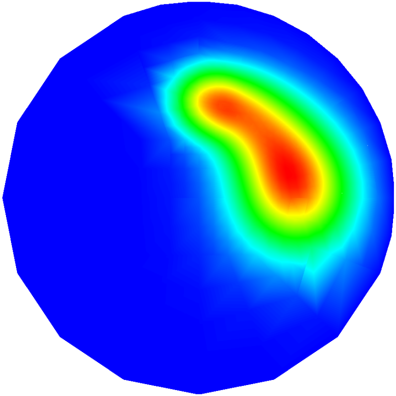
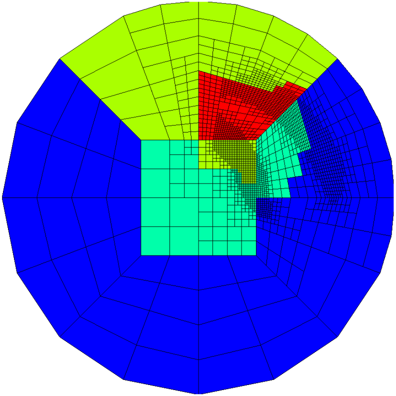


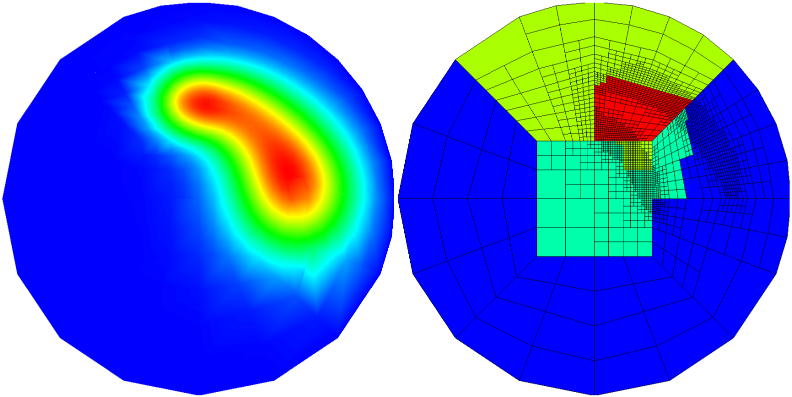


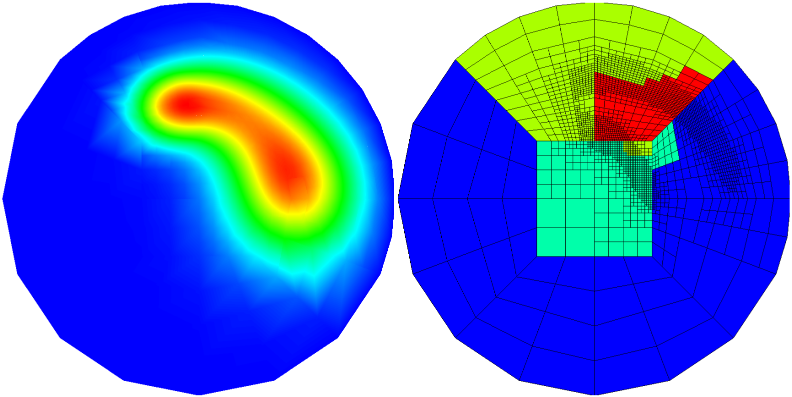


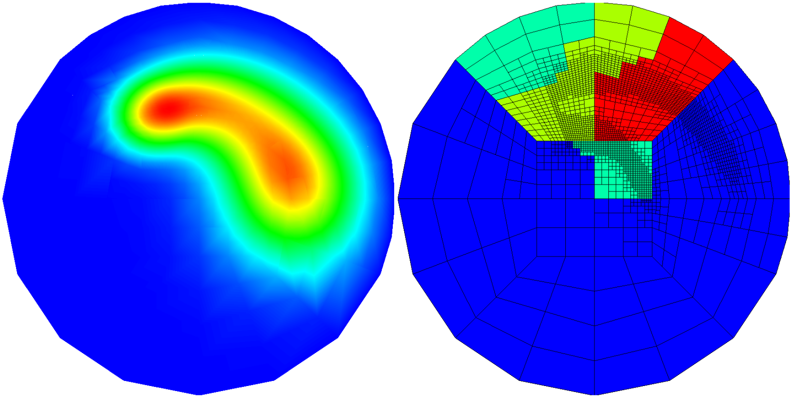


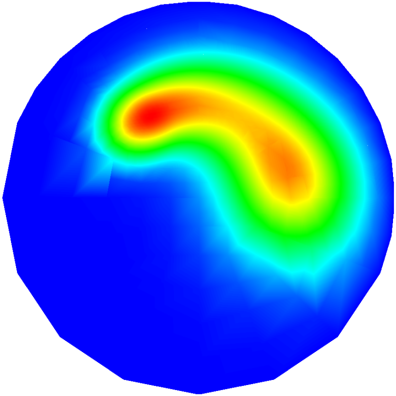


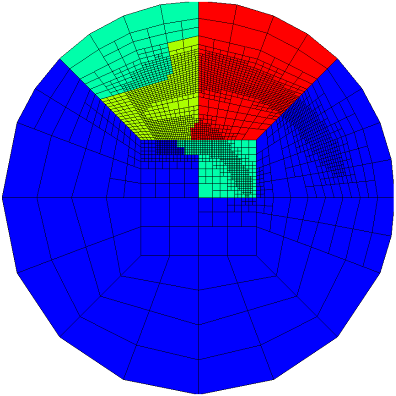



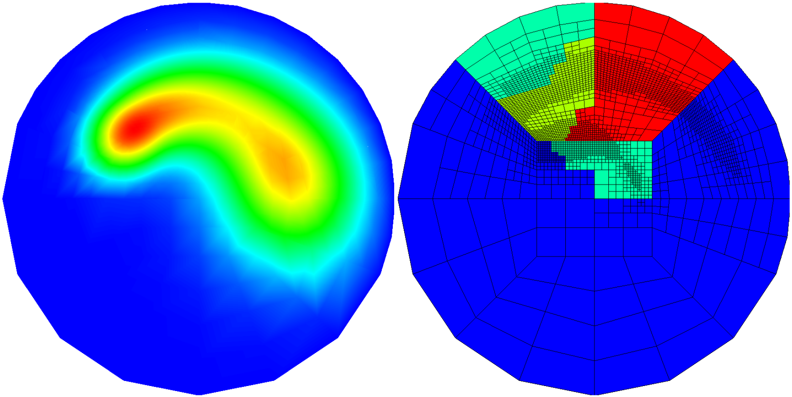


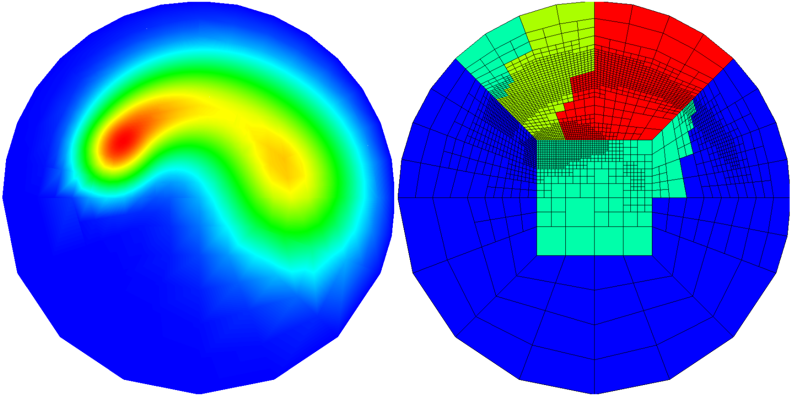


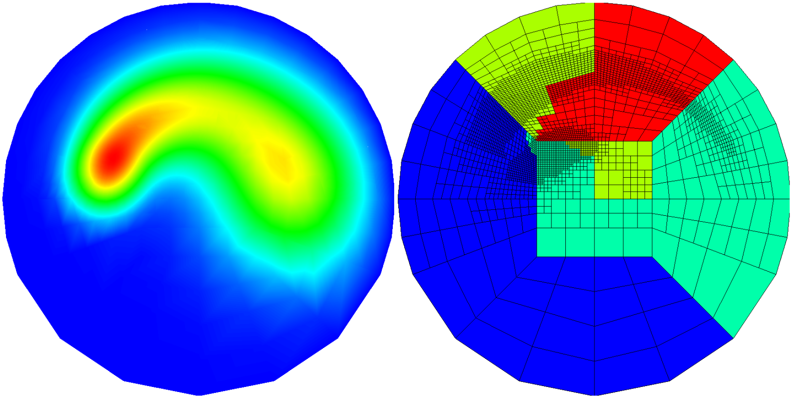


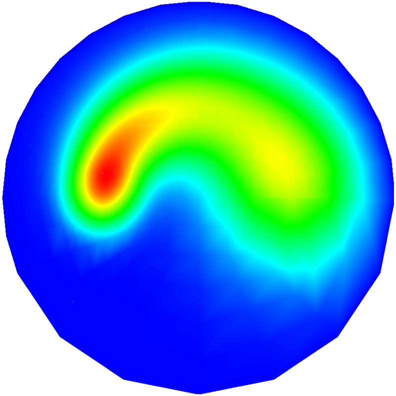


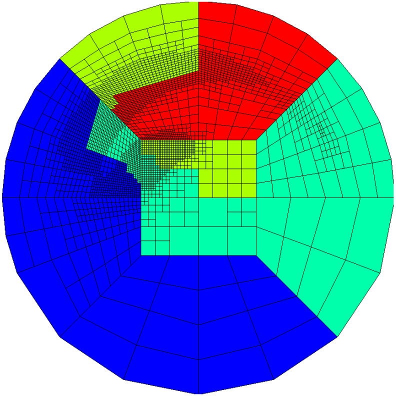


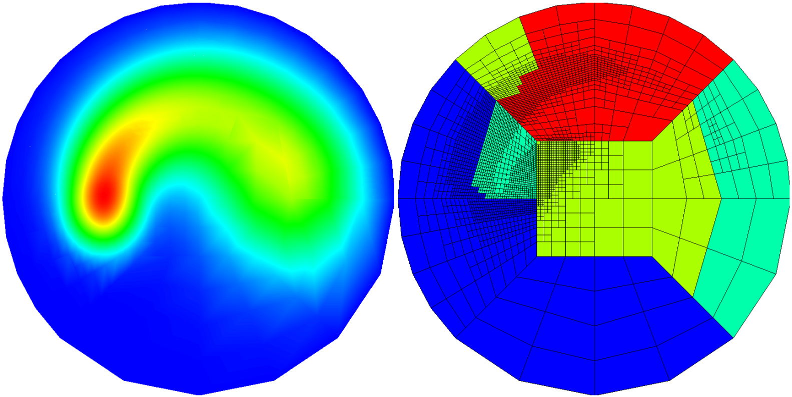


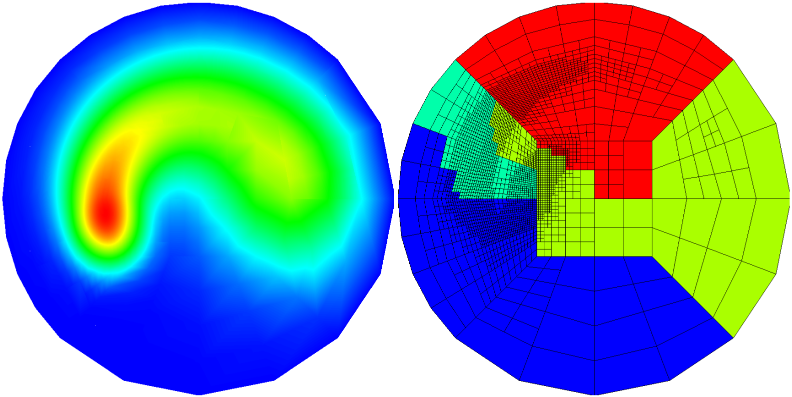


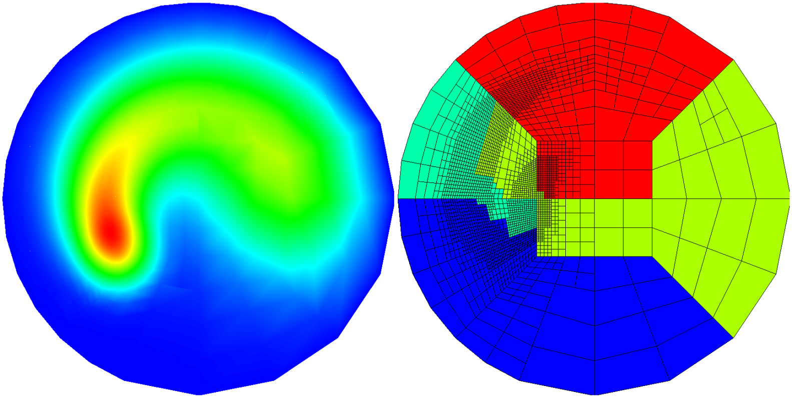


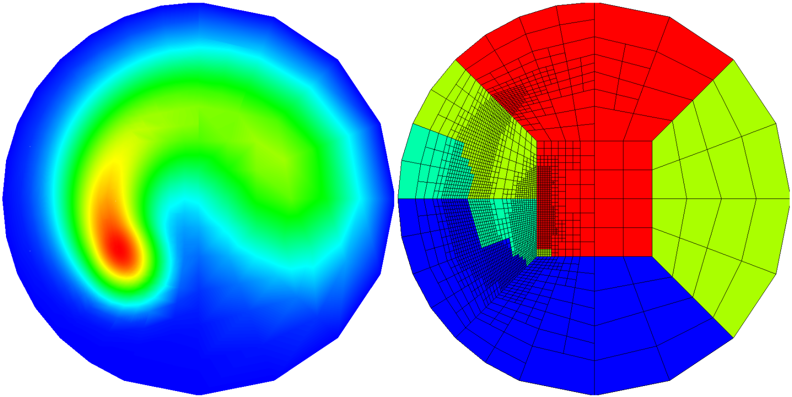


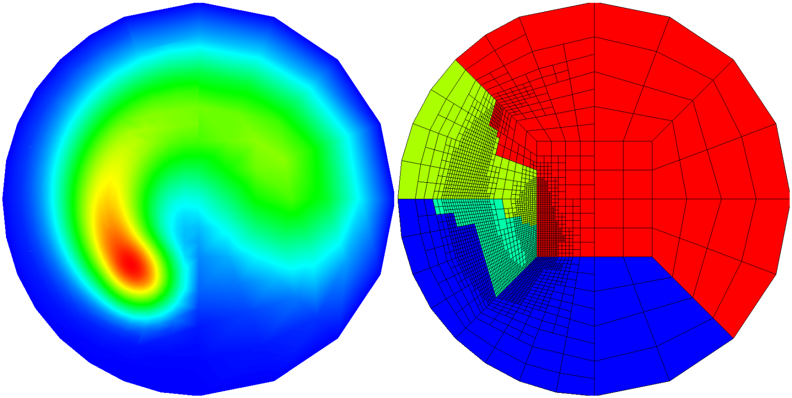


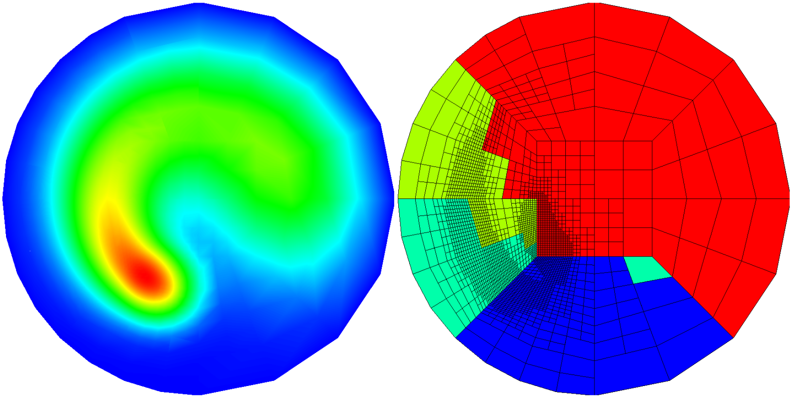


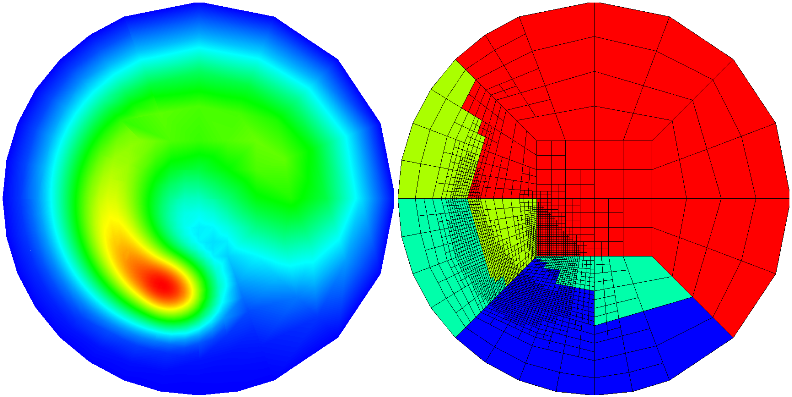


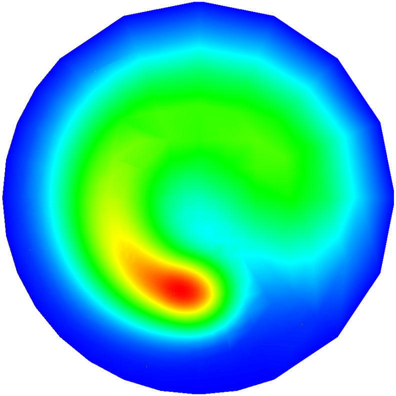


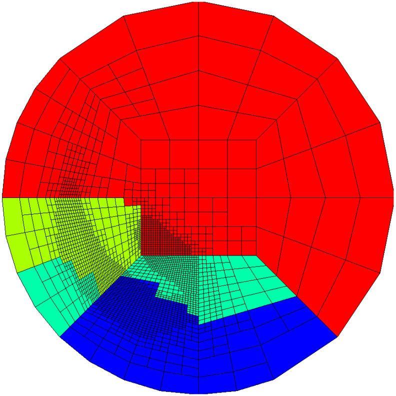




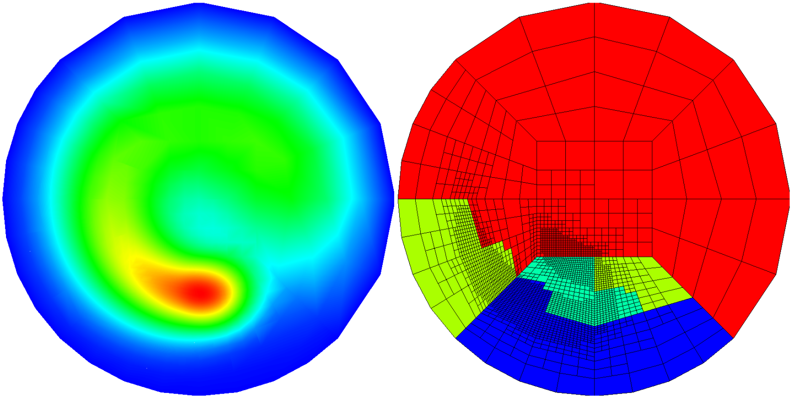


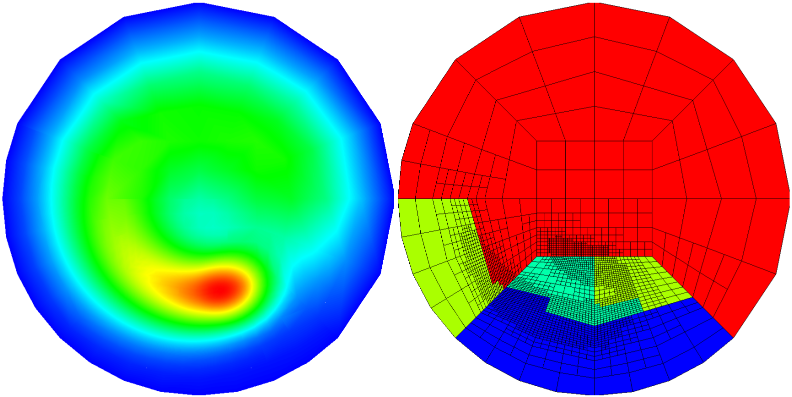


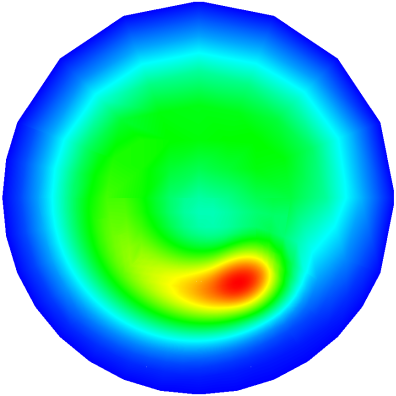


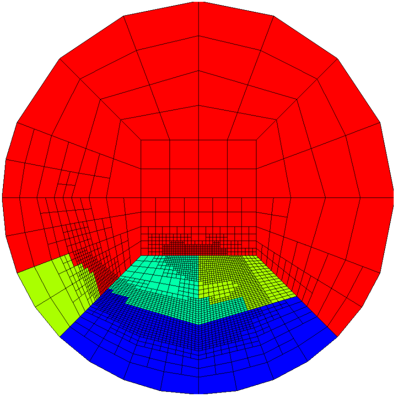


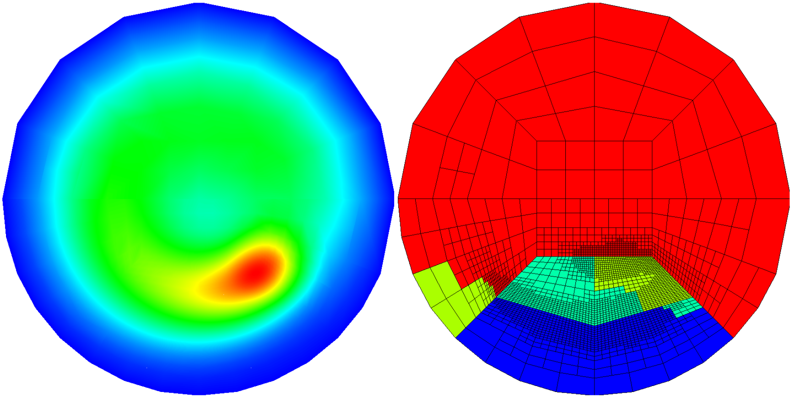
Member of the Helmholtz-Association

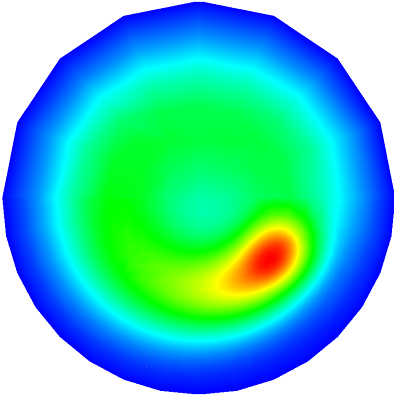


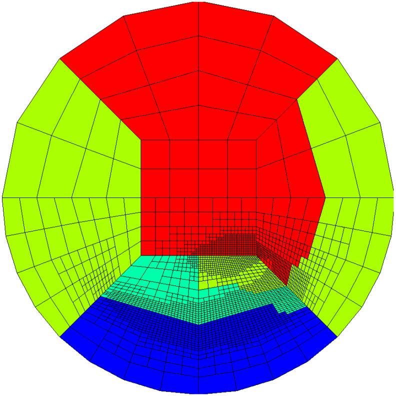


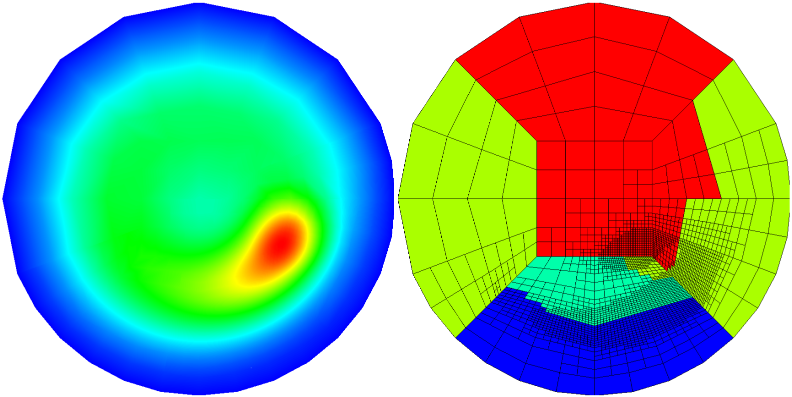


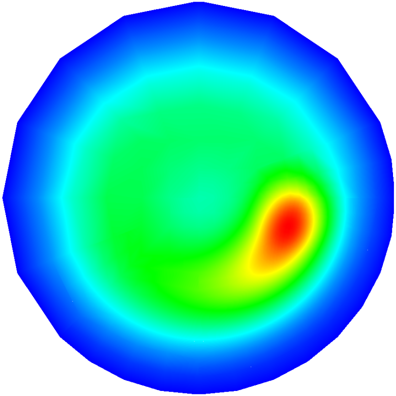


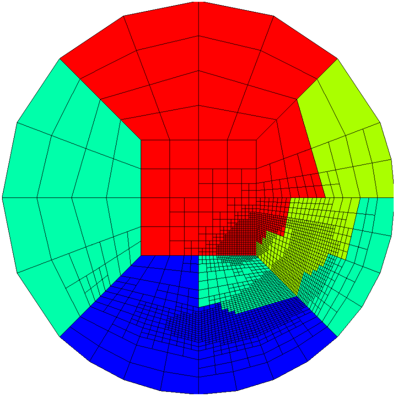


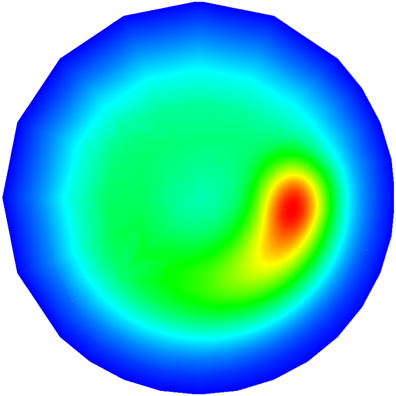


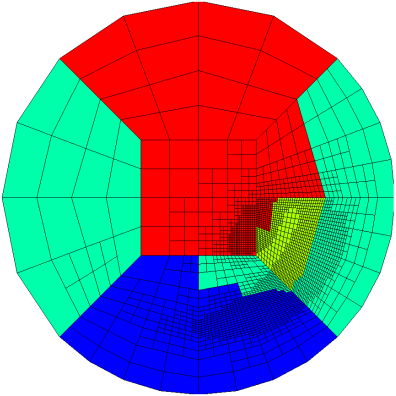


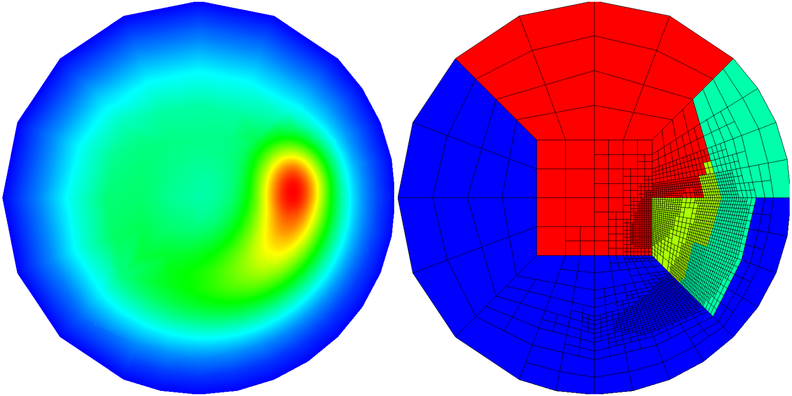


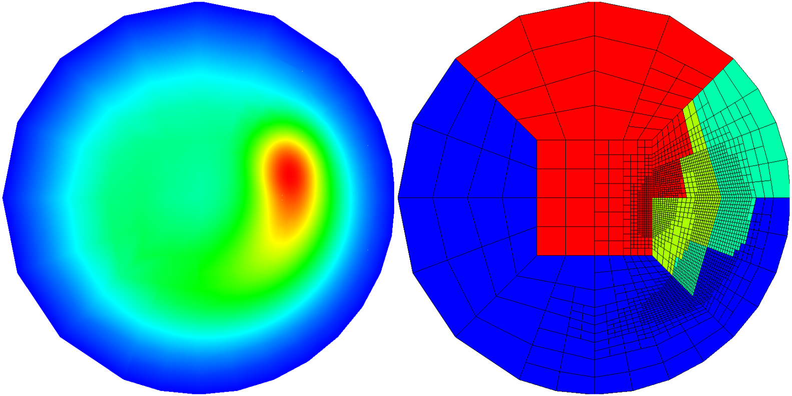


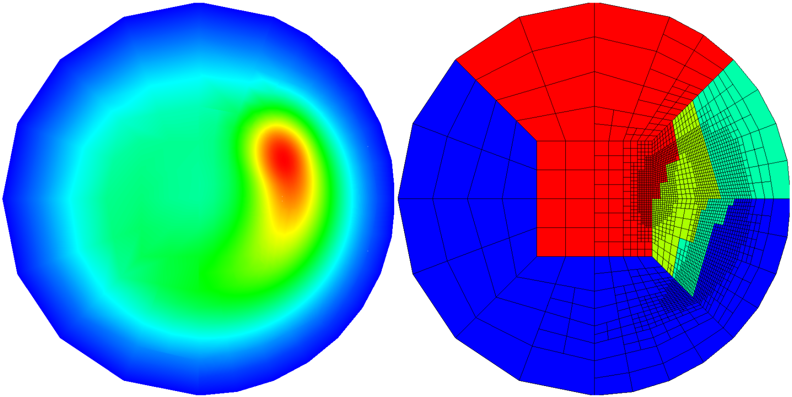


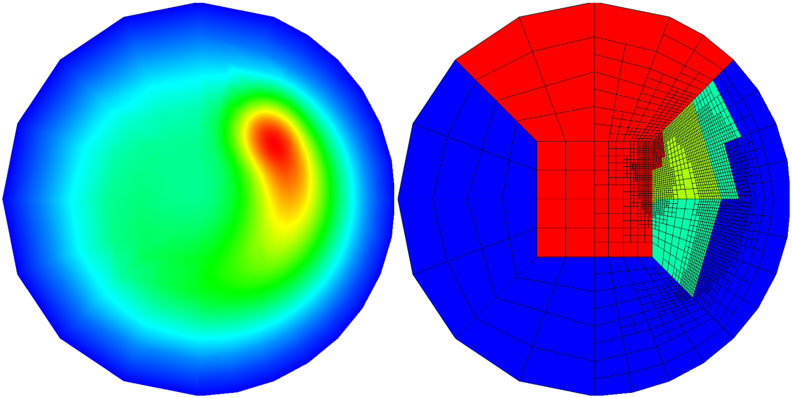


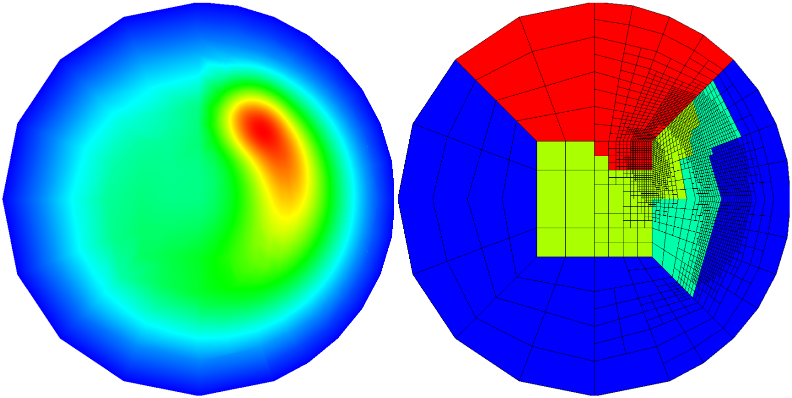


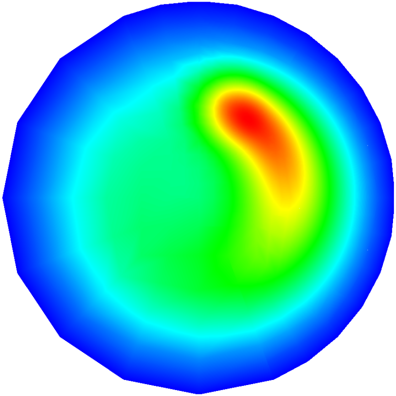


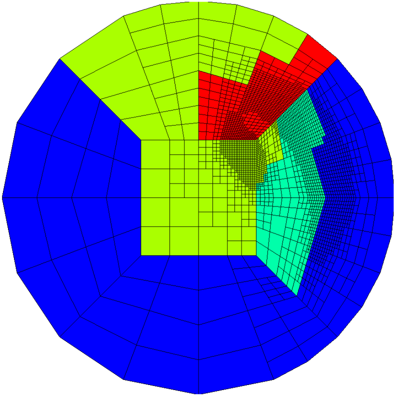












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FORSCHUNGSZENTRUM

dgfire

Part V: Future Work

April 8, 2014 | Kevin Drzycimski

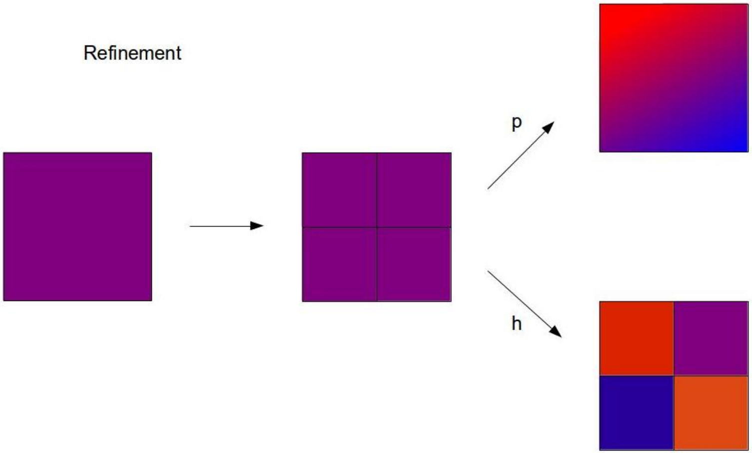
Future Work

hp-adaptive

- implement *hp*-Refinement for parallel use
- other refinement indicators?

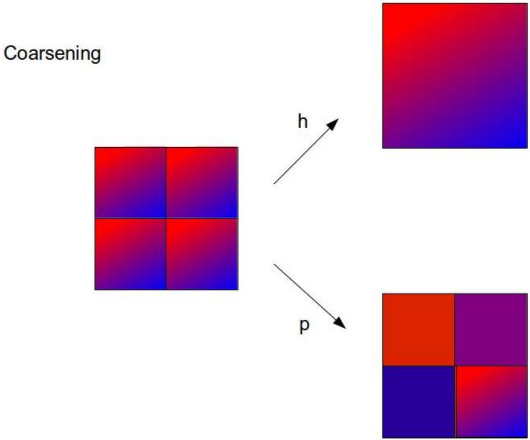
Future Work

hp-adaptive



Future Work

hp-adaptive

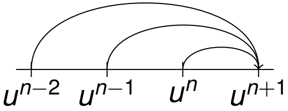


Future Work

Time stepping

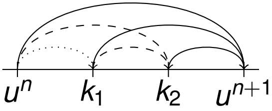
- Implicit Euler (1st order) not suitable for spatial high-order
- Need higher order time stepping scheme

BDF



+ only 1 Solution step
- not L-stable order > 2

Diagonally Implicit Runge-Kutta

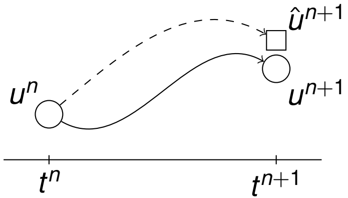


- multiple steps
+ L-stable
+ adaptive time stepping

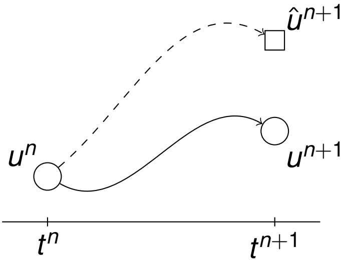
Future Work

Time stepping

- SDIRK4 is a 4th order scheme[1]
- also generates a 3rd order propagate $\hat{u}$ for free
- use $||u - \hat{u}||$ as temporal error indicator



solution accepted



discard solution
repeat with smaller timestep

[1] Hairer and Wanner, 1990

Future Work

within PhD thesis...

- more boundary conditions
- include density and multiple species
- verification

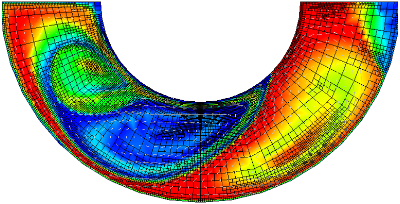
...and beyond

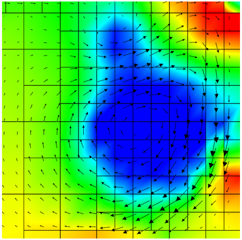
- radiation
- triggers, systems
- soot yield

The End

Thank you for your attention

Any questions?





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